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The Deflationary Bias of the ZLB and the FED's Strategic Response

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ABSTRACT

The paper shows, in a simple analytical framework, the existence of a deflationary bias in an economy with a low natural rate of interest, a Zero Lower Bound (ZLB) constraint on nominal interest rates and a discretionary Central Bank with an inflation mandate. The presence of the ZLB prevents the central bank from offsetting negative shocks to inflation whereas it can offset positive shocks. This asymmetry pushes average inflation below the target which in turn drags down inflation expectations and reinforces the likelihood of hitting the ZLB. We show that this deflationary bias is particularly relevant for a Central Bank with a symmetric dual mandate (i.e. minimizing deviations from inflation and employment), especially when facing demand shocks. But a strict inflation targeter cannot escape the sub-optimal deflationary equilibrium either. The deflationary bias can be mitigated by targeting "shortfalls" instead of "deviations" from maximum employment and/or using flexible average inflation targeting. However, changing monetary policy strategy risks inflation expectations becoming entrenched above the target if the natural interest rate increases.

Keywords: Monetary Policy Strategy, Inflation-Bias, Zero Lower Bound, Inflation Expectations.

JEL classification: E52, E58.

NON-TECHNICAL SUMMARY

The natural rate of interest (the real rate of interest which equilibrates the demand and supply of savings over the medium term) is estimated to have fallen persistently over the past decades in advanced economies. This fall in the natural rate has put severe limitations on the efficacy of conventional monetary policy. Given that nominal interest rates cannot go much below zero (the effective lower bound) and that a lower natural rate of interest implies lower nominal interest rate on average, this gives central banks less room to cut nominal interest rates to stimulate the economy. Additionally, if central banks cannot quickly reflate their economies, then inflation expectations will also fall. But nominal interest rates are composed of a real rate of return and expected inflation. Thereby, reducing inflation expectations hampers even further the room central banks have to deploy conventional monetary policy by lowering nominal interest rates. This constrains the central banks' ability to reflate the economy and drives down expected inflation even further, creating a vicious spiral. This adverse interaction between a low natural rate of interest and the effective bound is referred as the "deflationary bias". In response to this risk, central banks have deployed many unconventional monetary instruments but inflation has remained stubbornly below target (until very recently), dragging down inflation expectations.

Equilibrium inflation and interest rates under different monetary policy strategies
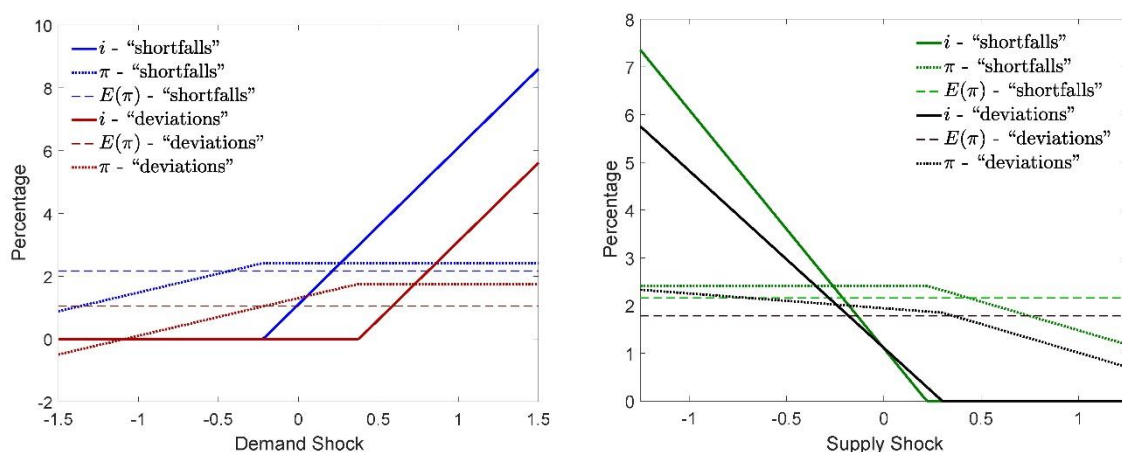
The Figure depicts different equilibrium interest rate, inflation and inflation expectations in the presence of demand (LHS) and supply (RHS) shocks for a Central Bank minimizing "deviations" or "shortfalls" of inflation and unemployment from their target.

In August 2020, the United States Federal Reserve announced two important changes to its long run goals and monetary policy strategy. It decided to shift its concern from "deviations" of employment from maximum employment to a concern for "shortfalls from maximum employment". Additionally, it has adopted a form of make-up strategy through the introduction of flexible average inflation targeting (FAIT).

In this paper, we first develop a simple model to explain the deflationary bias. To do this we extend the well-known Barro and Gordon (1983) model by adding a natural rate of interest and an effective lower bound at zero (ZLB). This modelling approach gives clear and intuitive analytical results and allows to compare the difference between a central bank with a dual mandate (on inflation and unemployment as for the Fed) or a pure inflation targeter (such as the ECB) and to study the effects of demand and supply shocks separately.

We demonstrate that in the presence of demand shocks, the deflationary bias is more severe for a central bank with a dual mandate than for an inflation targeting central bank. This is because the dual mandate central bank does not want to unduly overheat the labour market to increase inflation. We

also show that a flatter Phillips curve (less sensitivity of inflation to unemployment) reduces the deflationary bias, while a steeper IS curve (less sensitivity of unemployment to changes in nominal interest rates) exacerbates the deflationary bias. In the presence of supply shocks, the deflationary bias is less severe for a dual mandate central bank than for an inflation targeter: supply shocks move inflation and unemployment in opposite directions and the dual mandate central bank will tolerate higher inflation to reduce unemployment. This boosts inflation expectations and mitigates the deflationary bias.

Then, we show how switching from a "deviations" to "shortfalls" interpretation of the unemployment component of the dual mandate can mitigate and, indeed, overturn the deflationary bias. Under the ``shortfalls" strategy, the dual mandate central bank always prefers a lower rate of unemployment for any level of inflation. This leads to an upward bias in inflation and in the presence of the ZLB helps to combat the deflationary bias. Then we report simulations of the effect of switching to FAIT, a time varying temporary inflation target. We show that this strategy can mitigate the inflation bias but does not always eliminate it. This demonstrates the advantage of using both changes of strategy in tandem. In the final section of the paper we show that the welfare benefits of switching to "shortfalls" are positively correlated with the probability of hitting the ZLB.

Le biais déflationniste de la ZLB et la réponse stratégique de la FED

Résumé

L'article montre, dans un modèle analytique simple, l'existence d'un biais déflationniste dans une économie où le taux d'intérêt naturel est faible, où les taux d'intérêt nominaux sont soumis à la contrainte de la limite inférieure zéro (ZLB) et où la banque centrale a un mandat discrétionnaire en matière d'inflation. La présence de la ZLB empêche la banque centrale de compenser les chocs négatifs de l'inflation alors qu'elle peut compenser les chocs positifs. Cette asymétrie pousse l'inflation moyenne en dessous de la cible, ce qui entraîne une baisse des anticipations d'inflation et renforce la probabilité d'atteindre la ZLB. Nous montrons que ce biais déflationniste est particulièrement pertinent pour une Banque Centrale avec un double mandat symétrique (c'est-à-dire minimiser les déviations de l'inflation et de l'emploi), surtout lorsqu'elle fait face à des chocs de demande. Mais une cible d'inflation stricte ne peut pas non plus échapper à l'équilibre déflationniste sous-optimal. Le biais déflationniste peut être atténué en ciblant les « manques » au lieu des « écarts » par rapport à l'emploi maximum et/ou en utilisant une cible d'inflation moyenne flexible. Toutefois, en changeant de stratégie de politique monétaire, on risque d'ancrer les anticipations d'inflation au-dessus de l'objectif si le taux d'intérêt naturel augmente.

Mots-clés : Stratégie de politique monétaire, biais déflationniste, ZLB, anticipations d'inflation

Introduction

In August 2020, Jerome Powell, Chairman of the US Federal Open Market Committee (FOMC), announced two important changes to its long run goals and monetary policy strategy. The FOMC decided to shift its concern from "deviations" of employment from maximum employment to a concern for "shortfalls from maximum employment".[1] Additionally, it has adopted a form of make-up strategy through the introduction of flexible average inflation targeting (FAIT).[2]

We show in this paper how these changes in the Fed's monetary policy strategy help mitigate a deflationary bias that arises in an economy with a discretionary Central Bank (CB), a low natural rate of interest and a Zero Lower Bound (ZLB) constraint on nominal interest rates. Such an economy will have inflation expectations below the target, making the ZLB bind even more frequently and result in a sub-optimal equilibrium. We present a simple static model where this sub-optimal equilibrium arises and use it first to compare the role of different CB mandates. Then, we evaluate the changes in the Fed strategy by replacing a quadratic loss function on employment ("deviations") with a linear loss function ("shortfalls") and simulating a form of FAIT. We conclude with a welfare assessment of the change in strategy.

Our theoretical framework follows Barro and Gordon (1983) where inflation and unemployment are determined by two linear equations, a Phillips curve (henceforth PC) and an Investment-Saving relationship (IS). The model is closed using a monetary policy rule and assuming that the CB can use only the nominal interest rate to stimulate the economy (i.e. cannot use unconventional tools). The economy is perturbed by uniformly distributed i.i.d. demand and supply shocks, observed by the CB but not by economic agents before they make their choices. The CB has a specific mandate, known by everybody in the economy. The simplicity of the model is its strength. It allows us to us to derive analytical results and present them graphically in a very intuitive way.

We have four main findings. First, the deflationary bias is caused by the combination of a low natural rate of interest and a CB that cannot react to sharp falls in inflation because of the ZLB but also wants to keep inflation as close as possible to its target when it can, preventing significant positive deviations above its target. The deflationary bias is

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[1]On the Federal Reserve's website "maximum employment" is defined as "the highest level of employment or lowest level of unemployment that the economy can sustain while maintaining a stable inflation rate." In the setting of the Barro and Gordon (1983) model targeting "maximum employment" is equivalent to targeting the natural rate of unemployment.

[2]The adjective "flexible" is important because it only applies after episodes in which inflation has been below target and does not apply, as it would under average inflation targeting, for both overshoots and undershoots.

particularly acute in the face of demand shocks, mitigated by a flatter Phillips curve but amplified by a steeper IS curve. Second, the presence of a symmetric CB loss function focused on "deviations" amplifies the deflationary bias. Both a "shortfalls" and a FAIT strategy can help the Central Bank escape this sub-optimal equilibrium. Targeting "shortfalls" helps mitigate the pathology arising from the interaction between the symmetric dual mandate loss function and the ZLB and is equivalent to a *de facto* state-contingent increase in the inflation target. Similarly, FAIT raises the *de facto* inflation target but only when the ZLB does not bind. Third, either measure alone may not be sufficient but if the natural rate increases, "shortfalls" can become an unwelcome inflation bias. Fourth, moving to "shortfalls" strategy can be welfare enhancing when the probability of a binding ZLB is high.

We are not the first to show the possibility of a deflationary bias from the non-linear effect of the ZLB constraint but, to our knowledge, we are the first to explain how the central bank "deviations" strategy (i.e. the response to the Barro and Gordon (1983) inflationary bias) amplifies the bias. Thanks to our analytical solution, we can show very clearly where the bias comes from and how it differs between a central bank with a pure inflation mandate (such as the ECB) and a central bank with a dual mandate (such as the Fed). Additionally our framework allows to present how the problem depends on the prevalence of demand and supply shocks and on the structure of the economy (slope of the Phillips and IS curve). For example, the section on supply shocks allows to use our framework to check how the deflationary bias, and therefore the change in the FED strategy, can deal with the current potentially appearance of inflationary supply shocks.

Bianchi, Melosi and Rottner (2019) have also demonstrated the existence of a deflationary bias (which can degenerate into a deflationary spiral) from the possibility that monetary policy is constrained by the ZLB. This is the case even if monetary policy is not currently constrained. Similar to our mechanism, agent's expectations are the key. Bianchi, Melosi and Rottner (2019) present this in an extended New-Keynesian model which they solve numerically. They then show that an asymmetric policy strategy can eliminate the deflationary bias: responding less aggressively to positive deviations of inflation from the 2% target than to negative deviations, similarly to FAIT. Mertens and Williams (2019b) reach a similar conclusion using a simpler New Keynesian model augmented with a lower bound on interest rates (see also Mertens and Williams (2019a)). They evaluate the effect of standard inflation targeting (IT), average inflation targeting - including a make-up strategy - and a price level targeting on counteracting this bias. They find that dynamic rules, relying on history dependence, can eliminate the deflationary biases and deliver better macroeconomic outcomes than static rules (i.e. the Taylor

rule).

Mitigating the deflationary bias is probably not the unique explanation for the Fed's change of strategy. It likely also reflects changes in the way macroeconomists currently think about the business cycle. Indeed, the idea of a "cycle" that oscillates around a steady-state trend has been challenged. Dupraz et al. (2019) provide evidence for Milton Friedman's conjecture of a "plucking model" for the path of output relative to potential. Recessions are abrupt "shortfalls" of output from potential that is then made up during the recovery phase. Critically, deviations of output from potential are not symmetric: recoveries are linked to the preceding recessions but do no influence the depth of subsequent recessions.[3] A second important consideration is that there is no natural length to periods of economic expansion at close to potential growth. [4]

Our work is also related to the literature looking at the natural rate of interest (see, among others, Laubach and Williams (2015), Holston et al. (2017) and Brand et al. (2019)), the existence of a Zero Lower Bound and the deflationary bias. From Adam and Billi (2007) onward, most authors focused on how low natural interest rates affect optimal monetary policy (see, for example, Chung et al. (2012), Gust et al. (2017) and Andrade et al. (2018)) or how monetary policy could mitigate the problem arising from the ZLB. Kiley and Roberts (2017) find that based on a neutral nominal interest rate of three percent and a policy rule estimated from historical data, monetary policy in the US may be constrained by the ELB as much as one-third of the time. Bernanke et al. (2019) show that "lower-for-longer" policies can be quite successful in mitigating this risk, even with imperfect credibility.

In the construction of our analytical set-up, we borrow a lot from the literature using the Barro and Gordon (1983) model. Crucial insights are taken from papers assessing the role of credibility (Rogoff, 1985) or uncertainty (like Staiger et al. (1997), Estrella and Mishkin (1999) or Clouse (2018)). Additionally, as often is the case in these types of model, we work in a framework with multiplicity of equilibria and we need to make a choice on the type of equilibrium we want to focus on. Note, however, that our equilibria are not the same as Schmitt-Grohe et al. (2001), Benhabib et al. (2002) and Benhabib et al. (2003) among others based on the interaction between a Taylor rule and the ZLB.

The paper is structured as follows. Section (1) presents the model and then the reaction

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[3]Dupraz et al. (2019) find strong evidence that it is indeed the case in the Unites States since WWII.

[4]At the annual meeting of the American Economic Association in 2019 Janet Yellen said "I think it's a myth that expansions die of old age. I do not think that they die of old age. So the fact that this has been quite a long expansion doesn't lead me to believe that...its days are numbered", to which Ben Bernanke jokingly replied: "I like to say they get murdered." In this context, Lhuissier (2019) shows that the likelihood of a recession is not increasing in the length of the expansion. See also Diebold and Rudebusch (1999) and Rudebusch (2016).

to demand and the supply shocks separately. Section (2), using the theoretical framework, presents the modification introduced by the FOMC and analyzes their ability to fight the existing deflationary bias. Section (3) studies the welfare consequences of moving from a "dual mandate" to a "shortfalls" strategy. Finally, Section (4) concludes and discusses potential caveats.

1 The Model

Our building block is the Barro and Gordon (1983) model, including the real natural rate of interest. Aggregate supply and aggregate demand are represented by two linear equations:

$$u = u_N - a(\pi - \pi^e) - s \tag{1}$$

$$u = u_N + b(i - r^* - \pi^e) - d \tag{2}$$

Equation (1) is a standard new-classical Phillips curve with inflation, π, unemployment, u and inflation expectations, π^e.[5] u_N is the natural rate of unemployment. Equation (2) is an IS curve where i is the nominal policy instrument and r^* is the natural rate of interest. s and d are supply and demand shocks respectively where we use the conventional that a positive supply shock reduces inflation for any given unemployment rate, while a positive demand shock increases aggregate demand. We assume that s and d are independently, identically and uniformly distributed shocks over $[s_L, s_H]$ and $[d_L, d_H]$ respectively with $E[s] = E[d] = 0$ and that only one type of shock is observed at a time. This is purely for analytical and descriptive convenience.

A crucial assumption we make in line with Barro and Gordon (1983) is that the Central Bank will observe the shocks before it takes its decision on i but private agents do not. So private agents have to form their inflation expectations π^e based on the unconditional distribution of the shocks but knowing the mandate of the Central Bank, its optimal behaviour and the constraint it faces. The Central Bank can work out what π^e will be and therefore has control over feasible outcomes of unemployment and inflation whenever the ZLB doesn't bind.

Our innovation is to highlight the interaction between the natural rate of interest and the ZLB. In our baseline specification we assume the Central Bank seeks to minimise the

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[5]In the New Classical Phillips curve π^e can be interpreted as inflation expectations on today's inflation formed conditional on the information set available in the previous period (see, for example, Sargent and Wallace (1975)). The only information that agents have in this model are the unconditional distributions of the shocks, the mandate of the central bank and the structure of the economy.

following standard quadratic loss function:

$$L(s,d) = \frac{1}{2}\left(\pi - \pi^*\right)^2 + \frac{\lambda}{2}\left(u - u_N\right)^2 \tag{3}$$

subject to $i \geq 0$ and the Phillips curve, equation (1).[6] Thus, when $\lambda = 1$, we have the standard symmetric loss function used to derive the Barro and Gordon (1983) results.

1.1 The analytical solution

We start by sketching the generic solution method and delay to Appendix (A) all analytical derivations. For convenience, we focus here only on demand shocks (i.e. positive demand shocks are increasing aggregate demand).

It is a timeless model and the Central Bank makes a one-shot decision. This leads to a binary outcome. The Central Bank will minimise its loss function whenever the implied nominal interest rate is positive and otherwise set it to zero.

From the first order conditions of the problem we can establish that whenever the ZLB does not bind, the Central Bank wants to set

$$\pi_{NZLB} = \frac{a^2\lambda}{a^2\lambda + 1}\pi^e + \frac{1}{a^2\lambda + 1}\pi^* \tag{4}$$

with

$$i = r^* + \frac{a + b\left(a^2\lambda + 1\right)}{b\left(a^2\lambda + 1\right)}\pi^e - \frac{a}{b\left(a^2\lambda + 1\right)}\pi^* + \frac{1}{b}d \tag{5}$$
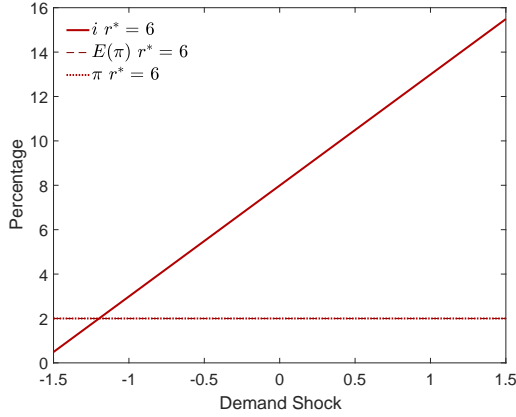
From this we can establish a threshold value of the shock $\bar{d}$, which identifies all the realization of d for which the ZLB will bind:

$$\bar{d} = \frac{a}{(a^2\lambda + 1)}\pi^* - \frac{\left[a + b\left(a^2\lambda + 1\right)\right]}{(a^2\lambda + 1)}\pi^e - br^* \tag{6}$$
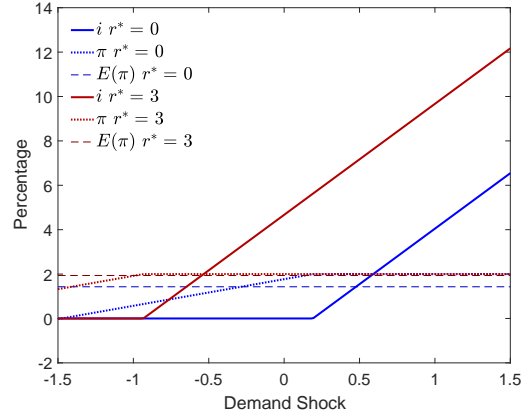
When the ZLB does bind, the Central Bank can do no better than set $i = 0$ and inflation will be

$$\pi_{ZLB} = \frac{(a + b)}{a}\pi^e + \frac{b}{a}r^* + \frac{1}{a}d \tag{7}$$

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[6]Equilibrium paths must also satisfy the IS curve, however, in the present case, the optimal paths of inflation and output can be determined without reference to the IS.

Figure 1: Equilibrium for inflation targeter for different values of r^*

Summarizing, this will lead us to the following simple CB decision rule:

$$\begin{cases} i = r^* + \frac{a+b\left(a^2\lambda+1\right)}{b\left(a^2\lambda+1\right)}\pi^e - \frac{a}{b\left(a^2\lambda+1\right)}\pi^* + \frac{1}{b}d \ \& \ \pi = \pi_{NZLB} = \frac{a^2\lambda}{a^2\lambda+1}\pi^e + \frac{1}{a^2\lambda+1}\pi^* & \text{if } d > \overline{d} \\ i = 0 \ \& \ \pi = \pi_{ZLB} = \frac{(a+b)}{a}\pi^e + \frac{b}{a}r^* + \frac{1}{a}d & \text{if } d \leq \overline{d} \end{cases}$$

We are now interested in finding the expected value of inflation, which will thus be the sum of the expected value of inflation when the ZLB does and does not bind:

$$E(\pi) = \int_{d_L}^{\overline{d}} \pi_{ZLB} \frac{1}{d_H - d_L}dd + \int_{\overline{d}}^{d_H} \pi_{NZLB} \frac{1}{d_H - d_L}dd \tag{8}$$

Expected inflation, π^e, of course will also appear on the right hand side of the expression. However, writing equation (8) in this format allows us to highlight the intuition of the solution as a fixed point problem - what value of π^e satisfies $E\pi = \pi^e$. Some algebra, detailed in the Appendix, transforms equation (8) into an inverted quadratic equation in either π^e or $\overline{d}$. In principle, there are two roots that satisfy this equation but only one of them is stable in an E-stability sense.[7] We will concentrate on this solution and the same logic will be used to solve all the subsequent equilibria. Details are available in the Appendix (A).

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[7]The concept of E-stability was first developed by DeCanio (1979), Evans (1985) and Evans (1986). Evans and Honkapohja (1992) provides an extensive survey of the literature.
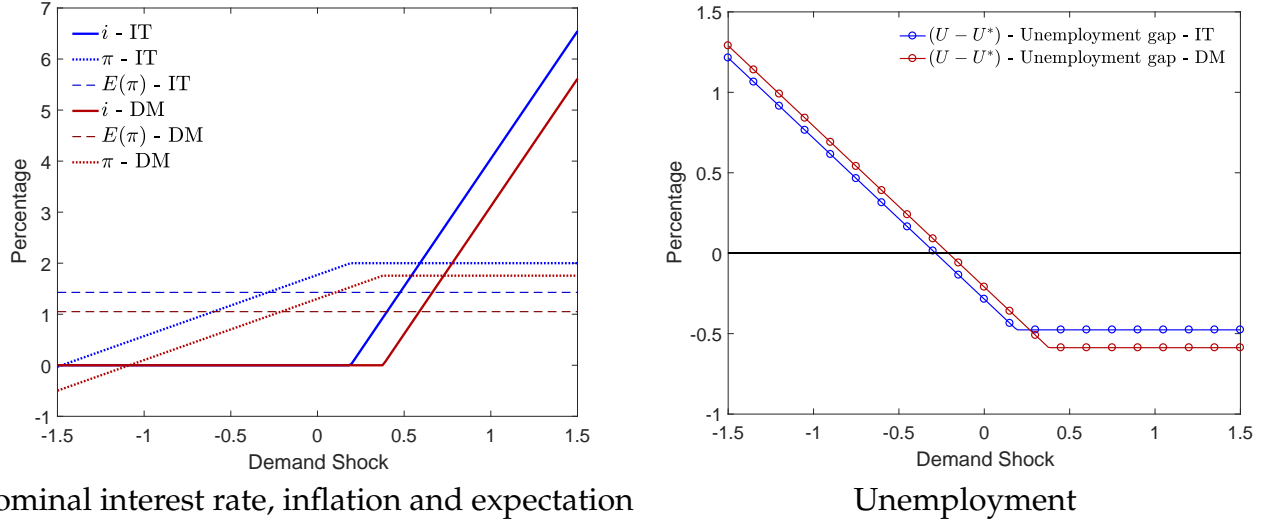
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Figure 2: Equilibrium for the Inflation Targeting vs. Dual Mandate

1.2 Demand Shocks

In this section we describe how the CB will respond to demand shocks. We first present the problem from the perspective of a pure inflation targeter, $\lambda = 0$ (i.e. assigning zero weight in the loss function to deviations of the output from its natural level). This simplifies the intuition. The left-hand panel of Figure 1 illustrates the trivial case in which the natural rate in the economy is sufficiently high, $r^* = 6\%$. In that case, the ZLB never binds for any value of the shock d considered, ranging from -1.5 to 1.5 (x-axis); the nominal policy rate i is therefore linear in d and $\pi = \pi^* = 2\%$ at all times. Obviously $\pi^e = \pi^*$ as well, and the lines $\pi = \pi^e = \pi^*$ overlap. Notice that this particular result would hold also for a Central Bank with a Dual Mandate. In fact the "divine coincidence" would apply and stabilising unemployment at u_N would be necessary, and sufficient, for stabilising also inflation at π^*.

The right-hand panel of Figure 1 illustrates what happens as r^* falls. The ZLB starts to bind for low realizations of d and π falls linearly as d becomes more negative because the CB cannot offset the impact of the shock. When the realised shock d_t is greater than $\bar{d}$, then the ZLB doesn't bind and the inflation targeter sets $\pi = \pi^*$. However, the possibility of a binding ZLB, as r^* falls, results in $\pi^e < \pi^*$ which drives down realized inflation during the ZLB phase - see equation (7) - and causes the ZLB to bind for higher values of d - see equation (6). This is the reinforcing nature of the ZLB equilibria and can be clearly seen from the drop in inflation expectations $E(\pi)$ in Figure 1 as r^* falls. We label this effect, as in the literature, the deflationary bias.

9

Figure 2 illustrates a key result of our analysis. It shows the equilibrium in the presence of demand shocks for a strict inflation targeting Central Bank ($\lambda = 0$) relative to that of a CB with a dual mandate, even if not attaching the same weight to both deviations ($\lambda = 0.5 > 0$). As mentioned before, if the ZLB never binds (r^* high), then the two equilibria are identical to the left-hand panel of Figure (1): the divine coincidence holds and the dual-mandate CB will stabilise both of its objectives at $\pi = \pi^*$. But this is no longer the case when the ZLB can bind occasionally. Both types of CB face the problem that $\pi^e < \pi^*$. Agents know that the inflation targeting Central Bank will respond by aggressively lowering i, resulting in $u < u_N$, in order to ensure that $\pi = \pi^*$ when the ZLB doesn't bind. The dual mandate CB, on the other hand, experiences a loss when $u < u_N$ and thus agents know that it trades off an undershoot of the inflation objective during non-binding times for smaller deviations from the unemployment objective. But this lowers π^e even further, causing the ZLB to bind at higher values of d, exacerbating the trade-off between its two objectives, and so on. The result is shown in the left-hand panel of Figure 2 in which actual and expected inflation, in both in ZLB and non-ZLB periods, are much lower for the dual mandate CB compared to the inflation targeting CB.
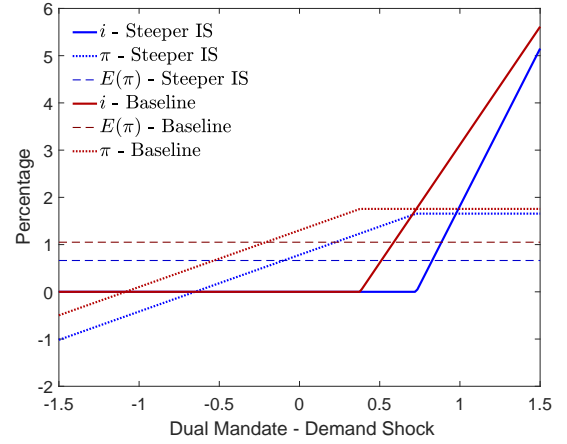
The right-hand panel of Figure 2 shows the effect of these equilibria on the labour market where the vertical axis reports $u - u_N$. This shows that despite having an unemployment objective, deviations from the natural rate of unemployment are larger for the dual mandate CB than the inflation targeting CB. Unemployment higher when the ZLB binds and lower when it does not. This is a general result. To understand why it is important to realise that average unemployment is equal to u_N - take expectations of equation (1). As a consequence, the integral of positive deviations must equal the absolute vale of the integral of negative deviations. Since π^e is lower for the dual mandate CB, the real interest rate is higher when the ZLB binds and also occurs for a larger range of values for d. Hence the unemployment rate must always be higher when the ZLB binds for both (as can be seen in the Figure). This must be compensated for by a lower unemployment rate when the ZLB doesn't bind for both.

The analysis of the two preceding paragraphs shows that a dual mandate CB with a "deviations" interpretation has unambiguously worse outcomes than an inflation targeting CB along both dimensions when the ZLB can occasionally bind. This situation only gets worse as the natural rate of interest falls.

In the remainder of this section we analyse the role of the slopes of the Phillips and IS curves on the deflationary bias in presence of a CB with a dual mandate. This is a detour from the main thread of the paper and can be skipped but we believe it is of interest and demonstrates the virtue of our simple set-up.
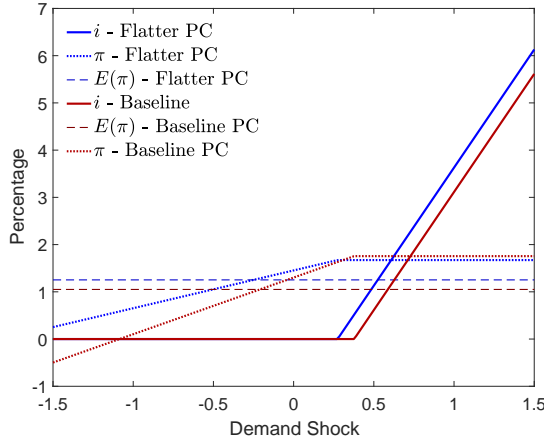
| Flatter Phillips Curve with Dual Mandate | Steeper IS curve with Dual Mandate |

Figure 3: Equilibrium for dual mandate under different slopes of the Phillips Curve

There is a common consensus that the PC has flattened, triggering an on-going debate on its cause and effects.[8] Excluding for a moment the presence of the ZLB, a flatter Phillips curve is unambiguously bad for a dual mandate CB because it worsens the trade-off between unemployment and inflation stabilisation in the presence of supply shocks, although makes no difference for demand shocks where the divine coincidence holds. The asymmetry created by the ZLB makes this more complicated. The inability to offset large negative demand shocks can actually result in a better equilibrium than in the presence of a steep Phillips curve (see Clouse (2018)). Similarly, the effect on the deflationary bias is not as trivial. The left-hand panel of Figure 3 highlights the consequences of a flatter Phillips curve on the economy with a CB with a dual mandate. Practically, this corresponds to a higher a parameter in equation (1). On the one hand, as mentioned above, a flatter Phillips curve worsens the trade-off for a Central Bank with the dual mandate when the ZLB doesn't bind because it is more costly, in terms of unemployment objective, to raise inflation back towards π^*. This matters now for demand shocks because expected inflation is no longer equal to π^* and the divine coincidence no longer holds. This results in lower inflation when the ZLB does not bind (the blue dotted line is below the red dotted line when the ZLB does not bind). On the other hand, however, inflation is considerably higher when the ZLB binds in the presence of a flatter Phillips curve because inflation does not fall by as much as negative demand shocks increase unemployment. By looking at inflation expectations, we can easily see that the latter effect always dominates

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[8]For a review of the current debate in the literature see Blanchard (2016), Del Negro et al. (2020), Hazell et al. (2020) and Siena and Zago (2021).

11

and inflation is unambiguously higher when the Phillips curve is flatter. In this particular set-up, therefore, the flatter Phillips curve is not a curse but helps mitigate the deflationary bias.

By contrast, a steeper IS curve unambiguously amplifies this bias. Over recent years, several economists have suggested that aggregate demand and output might have become less sensitive to interest rates (for example, Stansbury and Summers (2021) and Schnabel (2020)). In our analytical framework this would map into a fall in the parameter b in equation (2). The right-hand panel of Figure 3 shows the effect of such a change for a dual mandate CB. When the ZLB doesn't bind, the monetary authority needs to act more aggressively to maintain π close to π^* because π is indirectly less sensitive to changes in i. Since there is no upper limit to nominal interest rates, the central bank can always hit its preferred inflation rate when shocks are high. But when the IS curve is steeper and shocks are low, it runs out of room to manoeuvre much more quickly, making the ZLB bind over a wider range of demand shocks (i.e. more frequently), see Figure 3, right-hand panel. This is a reinforcing problem. When the ZLB binds, the CB cannot offset the effect of those demand shocks and inflation falls, dragging down inflation expectations which as in the baseline analysis makes the ZLB bind more frequently and so on. But lower inflation expectations also cause the CB to set a lower inflation rate when the ZLB doesn't bind because otherwise it will have to drive u even further below u_N to achieve the same inflation result which it will not want to do. This lower level of inflation when the ZLB doesn't bind, amplifies the deflationary bias.

1.3   Supply shocks

We now turn our attention to supply shocks. Figure 4 reports the results for a CB with an inflation target and with a dual mandate. By our definition of supply shocks, the ZLB will bind following positive supply shocks that push down inflation for any given unemployment rate. In our one-shot scenario, the CB will want to run a more accommodative policy to bring inflation closer to target when supply shocks are positive. The ZLB therefore binds for high realizations of the shock (the right hand side of the x-axis).

As might be expected, the equilibrium for the inflation targeting CB under supply shocks is a mirror-image of the demand shock. When the ZLB doesn't bind, the inflation targeting CB sets $\pi = \pi^*$ and faces a linearly declining inflation rate when the ZLB binds. As in the demand shock case, the inflation targeting CB is subject to a deflationary bias.

The dual mandate Central Bank, by contrast, is less aggressive in combating negative supply shocks because lowering inflation would require a more negative unemployment

gap. So the more negative the shock, the more inflation the dual mandate Central Bank is willing to tolerate. As a consequence, the slope of the reaction function is less steep for the dual mandate CB when the ZLB doesn't bind. This higher inflation when the ZLB doesn't bind increases inflation expectations relative to the inflation targeter which in turn reduces the frequency of the ZLB constraint and raises the inflation rate when the ZLB binds. Overall, inflation expectations are higher for the dual mandate Central Bank than the inflation targeter in the presence of supply shocks. However, for both types of Central Bank, inflation expectation are below π^* and the deflationary bias is still in place. As a result, the variance of unemployment is higher for the inflation targeter when the ZLB does not bind. This is the case as the inflation targeter does not allow supply shocks to move inflation, resulting in higher unemployment fluctuations. By contrast, the dual mandate Central bank smooths unemployment movements by allowing inflation to increase in response to negative supply shocks. However, interestingly, when the ZLB is binding, the inflation targeter will have a smaller unemployment gap. This is a direct result of higher average inflation expectations for the dual mandate central bank (see the left-hand panel). From the IS equation, one can see that unemployment at the ZLB will be a constant determined by the real interest when the nominal interest rate is zero net of the natural rate of interest. This is the negative of low expected inflation. The dual mandate CB has higher inflation expectations and thus a lower real rate at the ZLB which translates into a more negative unemployment gap.

Therefore, the presence of the ZLB and a low natural interest rate are unambiguously bad for inflation expectations for both an inflation targeting CB and a dual mandate CB. There is a deflationary bias in all cases. Which type of CB is worse affected depends on the relative frequency of demand versus supply shocks.

2 FOMC amendments to their monetary policy strategy

The key lesson that CBs learnt from the Barro and Gordon (1983) model was that if they were concerned to stabilise employment (or unemployment), they had to do so symmetrically about the natural rate of unemployment. Trying to limit average unemployment over time would not work and would eventually lead only to higher average inflation. This symmetric concern for unemployment was embodied in the FOMC's original long term goals and strategy statement through the objective to limit "deviations of employment from the Committeeâs assessments of its maximum level", with maximum employment understood as corresponding to the natural rate of interest. This symmetry was crucial in giving credibility to an inflation target when the ZLB never binds and discre-
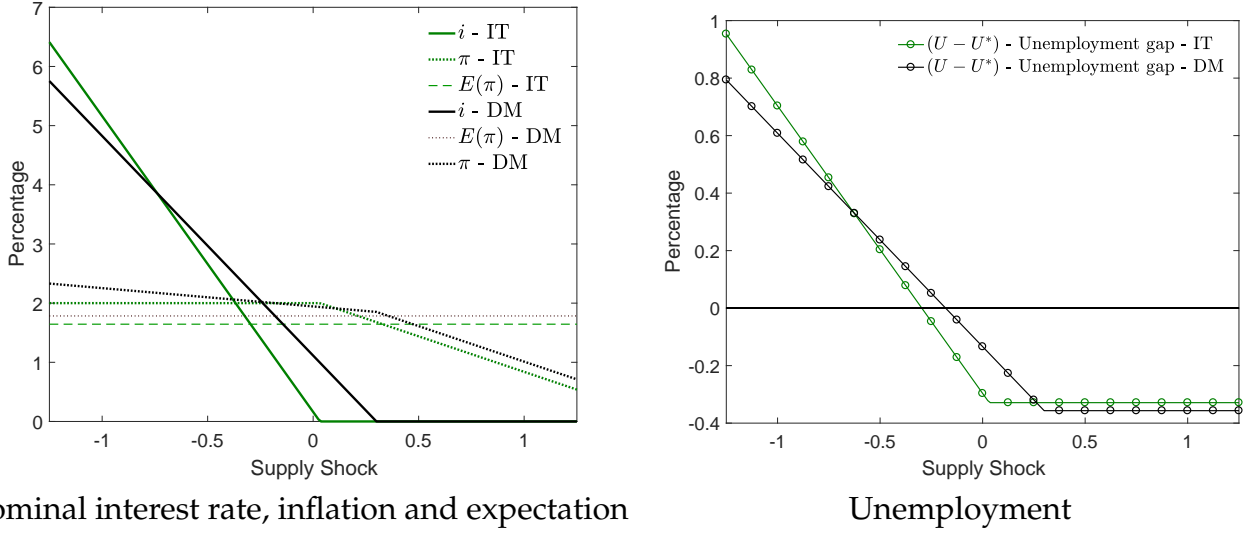
Figure 4: Equilibrium for the Inflation Targeting vs. Dual Mandate with Supply Shocks

tionary policy created an upward inflation bias.

The foregoing analysis, however, demonstrated that a symmetric concern for unemployment amplifies the deflationary bias created by the presence of the ZLB. In August 2020, the FOMC announced two important changes to its long term goals and monetary policy statement to counteract this deflationary bias. First, it replaced reference to concerns about "deviations" in employment from its maximum level to "shortfalls from maximum employment". Second, it introduced a flexible make-up strategy based on occasional average inflation targeting. We consider the implications of each change in turn within the context of the model presented in the previous section.

2.1 Shortfalls

To model the switch from "deviations" to "shortfalls" we adjust the CB loss function to be:

$$L(s,d) = \frac{1}{2}\left(\pi - \pi^*\right)^2 + \phi\left(u - u_N\right) \tag{9}$$

(Note that the weighting ϕ has no necessary relationship to λ, the weight on the quadratic unemployment term.) This is the simplest adjustment to the loss function consistent with the shift to "shortfalls". We will discuss concerns about this formulation after presenting the results. We assume implicitly that this change of strategy is completely credible and that the central bank does not suffer from a time-inconsistency problem. In practice, of course, this may be harder to achieve and it may be costly for the central bank (in terms

of worse outcomes on unemployment and inflation) to demonstrate its commitment to its new strategy.

As might be expected, removing the quadratic term in unemployment induces an upward bias in inflation when the ZLB doesn't bind (by exactly $a\phi$). It is observationally equivalent (for demand shocks) to a symmetric loss function with a higher value for π^*. The resulting equilibrium is illustrated in the left-hand top panel of Figure 5. This has a powerful impact. Not only is inflation higher when the ZLB doesn't bind, the resulting increase in expected inflation drags up inflation when the ZLB does bind and shifts the ZLB threshold to the left. These elements reinforce each other. Using "shortfalls" instead of "deviations" is thus a way of introducing a *de facto* higher inflation target without changing it *de jure*. Expected inflation is a continuously increasing function of ϕ, the weight the central bank puts on reducing unemployment. Through an appropriate choice of ϕ, the dual mandate Central Bank could ensure that $\pi^e = \pi^*$. In this way, the central bank can offset one bias with another, although the equilibrium is not the same as if the ZLB was not a constraint. In principle, therefore, shifting to a "shortfalls" strategy is all that would be necessary to inflation on average at target in the presence of demand shocks only.

It's also interesting to see the effects of shifting to a "shortfalls" strategy in the presence of supply shocks. The strategy was conceived in the context of limits to stimulating demand but must now deal with potentially inflationary supply shocks. The right-hand top panel of Figure 5 shows that average inflation is higher under the "shortfalls" strategy because the central bank is willing to tolerate higher inflation to prevent unemployment rising too far above the natural rate of unemployment. However, inflation does not increase as supply shocks worsen because the central bank implicitly targets a constant inflation rate when the ZLB doesn't bind. This is because under "shortfalls" the marginal cost of additional unemployment is constant but increasing in inflation. So the CB will implicitly target a constant inflation rate when the ZLB doesn't bind. This strategy requires the CB to be more aggressive in counteracting negative supply shocks. So in this instance, even though inflation expectations are higher, the ZLB actually binds over a wider range of shocks.

The bottom panels of Figure 5 report the effects of the shift from "deviations" to "shortfalls" on unemployment. Recall that average unemployment must equal the natural rate of unemployment in rational expectations equilibrium. The bottom left-hand panel reports the counter-intuitive result that the unemployment gap is smaller under "shortfalls" than under "deviations". It is notable that the undershooting of unemployment is markedly smaller, even though this is now a welfare gain for the CB. This occurs because
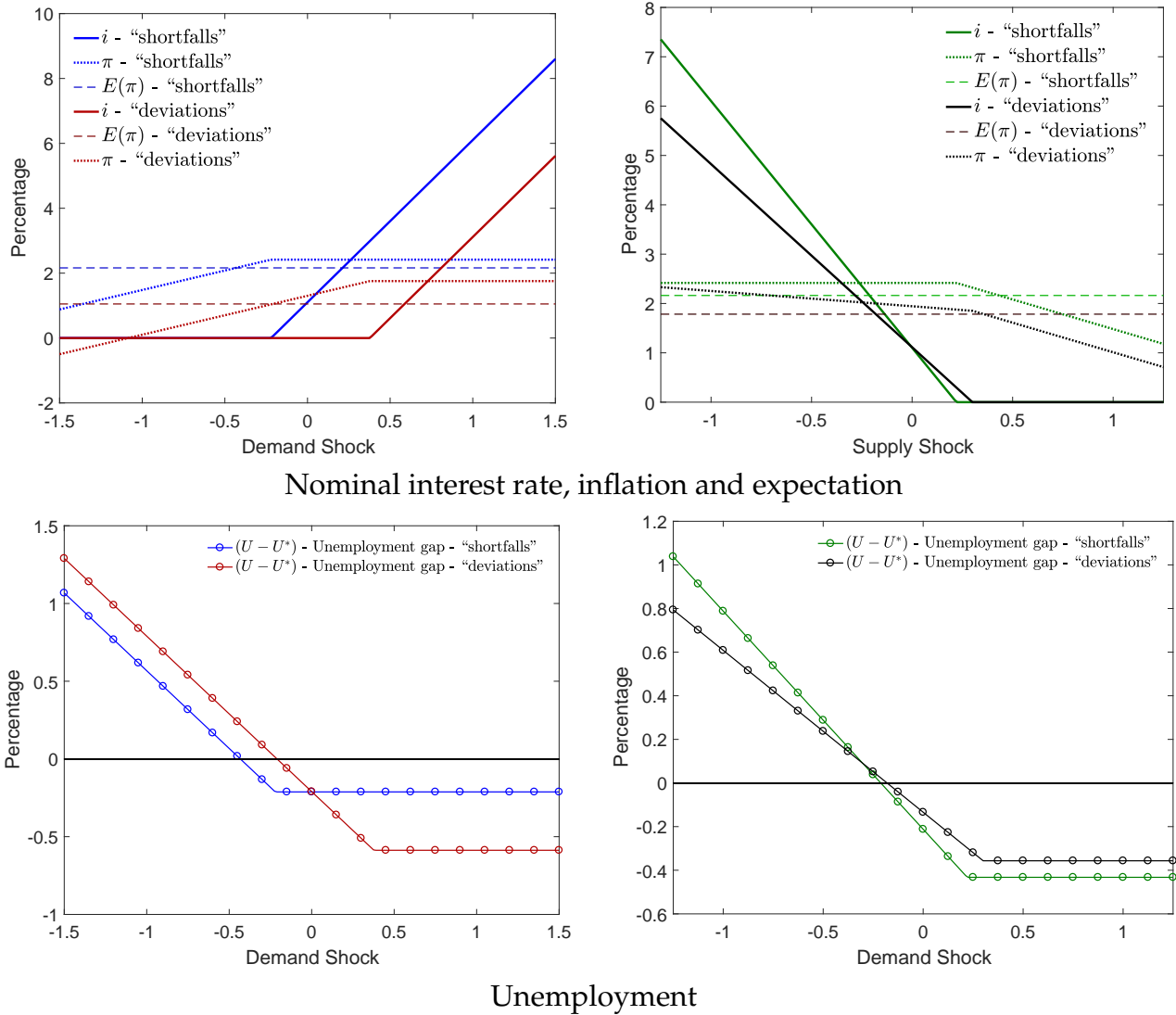
Figure 5: Effect of shifting from "deviations" to "shortfalls"

of the increase in inflation expectations which reduces the need to undershoot on un-
employment to get inflation above the target. This result is important because it shows
that the CB doesn't have to be more "reckless" on unemployment to make the "shortfalls"
strategy work. The situation is much less favourable in the presence of supply shocks
(right-hand bottom panel of Figure 5). In this case, unemployment fluctuations are wider
under "shortfalls". This occurs because under "shortfalls", the CB no longer mitigates the
effect of negative supply shocks on unemployment by allowing a higher inflation rate.

There might be some concern that our results depend in some way on the incentive
to lower unemployment indefinitely. Would this not be non-credible at a certain point?
What would happen, in other words, if there was an additional constraint in the CB's

optimisation problem that $u \geq \bar{u}$ for some $\bar{u} \leq u^*$? The bottom left-hand panel of Figure 5 shows that this concern is unfounded as long as the constraint is not too tight. Under the "shortfalls" strategy, the CB sets a constant unemployment rate when the ZLB doesn't bind. Provided $\bar{u}$ is below this unemployment rate, then the constraint is always slack and doesn't alter the results. And as noted above, the equilibrium "overheating" in the labour market is actually smaller than under the "deviations" strategy and so would not risk being seen as implausible. Nevertheless, if $\bar{u}$ were to be set "too tight", then the constraint would always bind when the ZLB did not. In this case, the CB would be a *de facto* unemployment targeter, the implications of which we will not consider here.

2.2 Make-up Strategy

The second change made by the FOMC was to introduce FAIT. If inflation persistently falls below its target because of hitting the ZLB, the FOMC will deliberately target a higher inflation rate in the future to compensate. The FOMC has been deliberately vague about the parameters of FAIT. To analyse this change we modify the loss function to

$$L(s,d) = \frac{1}{2} \left(\pi - \hat{\pi}_t^* \right)^2 + \frac{\lambda}{2} \left(u - u_N \right)^2 \tag{10}$$

where $\hat{\pi}_t^*$ will depend on past realisations of π.

Our very simple model has no dynamics and therefore is not particularly well-suited to a complete analysis of average inflation targeting. However, its simplicity does allow us to consider a simulation in which the Central Bank adopts FAIT. We will use the following simple algorithm:

- for $t = 1$ a shock from the uniform distribution will be drawn and $\pi(1)$ will be solved for under the assumption of $\hat{\pi}_1^* = \pi^* = 2\%$

- if $\pi(1) > \pi^*$, then $\hat{\pi}_2^* = \pi^*$

- if $\pi(1) < \pi^*$, then $\hat{\pi}_2^* = 2\pi^* - \pi(1)$

- and so on for $t = 2$ to T

The results of this exercise are presented in Figure 6. The upper left-hand panel illustrates average inflation from the start of the simulation to that point for demand shocks for $r^* = 0$ and $r^* = 0.75$. We run a thousand simulations and report the 10th percentile and 90th percentile of the range. Below this are reported the respective expected values of inflation in the equivalent baseline case and the average inflation rate across the sim-

17
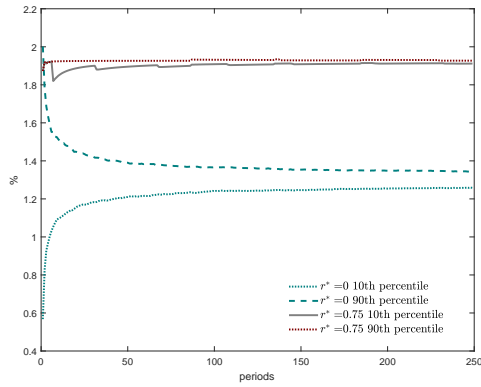
Demands shocks
$\pi^e \, (r^* = 0) = 1.05$
$\bar{\pi}(r^* = 0) = 1.32$
$\pi^e \, (r^* = 0.75) = 1.50$
$\bar{\pi}(r^* = 0.75) = 1.92$
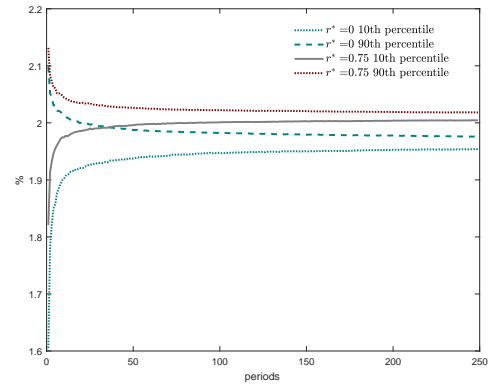
Supply shocks
$\pi^e \, (r^* = 0) = 1.78$
$\bar{\pi}(r^* = 0) = 1.97$
$\pi^e \, (r^* = 0.75) = 1.92$
$\bar{\pi}(r^* = 0.75) = 2.01$

Notes: Lines represent the interdecile ranges for simulations of flexible average inflation targeting (FAIT) using the algorithm in the text. π^e is the timeless solution for expected inflation without FAIT. $\bar{\pi}$ is average inflation over the full simulation across the whole cross-section. The gap between the two is a measure of the increase in inflation generated by FAIT.

Figure 6: Effect of shifting to flexible average inflation targeting

ulations across the whole period. The panel on the right shows the same results for the simulation with supply shocks.

Numerous results are apparent. FAIT raises average inflation towards π^* in all cases. It can completely overcome the downward bias from the presence of the ZLB in the case of supply shocks, although there was comparatively little to make up. It also works quite successfully for our simulation with $r^* = 0.75$, raising average inflation from 1.5 to over 1.9. However, it cannot compensate for the ZLB when $r^* = 0$ and it also takes much longer for this simulation to converge to its long run average. When $r^* = 0$ under demand shocks, π is frequently less than π^* but even though the CB subsequently targets a higher inflation rate, the risk of the ZLB binding is still sufficiently high that inflation expectations remain well below π^*. The CB cannot bootstrap its way back to π^* just through FAIT.
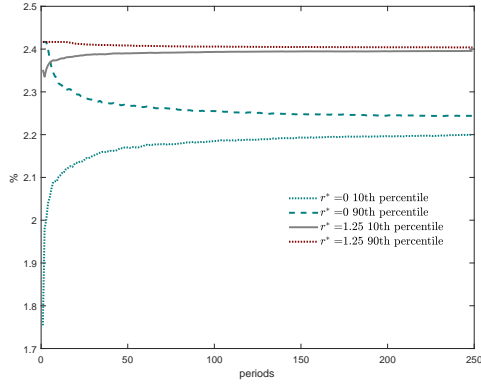
A similar result occurs if the Central Bank announces a modestly higher inflation target (of course, these results are purely illustrative). Inflation expectations and average realisations will increase but not by enough to escape the ZLB sufficiently often that average inflation will be close to the new target. Arguably Japan is in this equilibrium.

Finally, for completeness, we show both strategies together. Figure 7 reports the same type of simulations as Figure 6 but jointly with a "shortfalls" strategy. This clearly increases inflation even further for the case of $r^* = 0$ and for both demand and supply shocks, inflation is well above π^*. The second set of simulations assume $r^* = 1.25$. At this point the economy has essentially escaped the ZLB so the switch to "shortfalls" and the addition of flexible inflation targeting adds little extra.

3   Welfare implications

Without any presumption of being able to quantify the actual benefits, we exploit the simplicity of the model to study welfare implications of moving from a "deviations" to a "shortfalls" strategy. From the previous analysis we saw that the change in strategy raises inflation expectations and makes the ZLB binding less frequently, conditionally on the structure of the economy (e.g. slope of the Phillips and IS curve, π^* and r^*, the policy goal, etc). We now want to assess under what conditions this change can indeed be welfare enhancing.

We construct a micro-founded utility-based welfare measure and use it to evaluate the gains from adopting a "shortfalls" strategy. Following Rotemberg and Woodford (1997), we use the level of expected utility as a natural welfare criterion and compute a second-order Taylor approximation of it for a representative household in a rational expectations
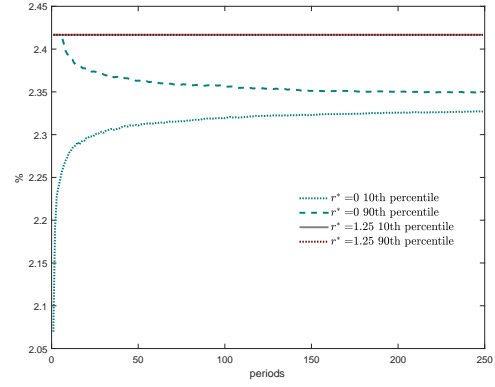
Demands shocks	Supply shocks
$\pi^e\ (r^* = 0) = 1.99$	$\pi^e\ (r^* = 0) = 2.16$
$\bar{\pi}(r^* = 0) = 2.22$	$\bar{\pi}(r^* = 0) = 2.35$
$\pi^e\ (r^* = 1.25) = 2.23$	$\pi^e\ (r^* = 1.25) = 2.33$
$\bar{\pi}(r^* = 1.25) = 2.40$	$\bar{\pi}(r^* = 1.25) = 2.42$

*Notes:*Lines represent the interdecile ranges for simulations of flexible average inflation targeting (FAIT) using the algorithm in the text. π^e is the timeless solution for expected inflation without FAIT. $\bar{\pi}$ is average inflation over the full simulation across the whole cross-section. The gap between the two is a measure of the increase in inflation generated by FAIT.

Figure 7: Effect of shifting to flexible average inflation targeting with shortfall strategy

setup $E\left\{\sum_{t=0}^{\infty}\beta^t U_t\right\}$. In the case of the New Classical model described above, the utility flow of the household each period can be approximated with:

$$U \approx \Omega L$$

where Ω is a positive constant (we set it equal to $\frac{1}{2}$) and L_t is a quadratic loss function:

$$L^{\text{Welfare}} = (\pi - \pi^e)^2 + \omega\ (u - u_N)^2 \tag{11}$$

Despite its simplicity, this welfare measure has the advantage of giving us a precise indication on the relative welfare gain of moving to a "shortfalls" strategy. As we have seen in section 2, the different policies lead to different inflation and unemployment realization, allowing us to evaluate the loss corresponding to each strategy. Figure 8 summarizes the result of this analysis for demand shocks. We have chosen to normalise the comparisons through the probability of hitting the ZLB. Keeping the parametrization presented in the previous sections, we increase the bounds of the demand and supply shocks to map a ZLB probability from 10% to 55%. This measure is closely related to the variance
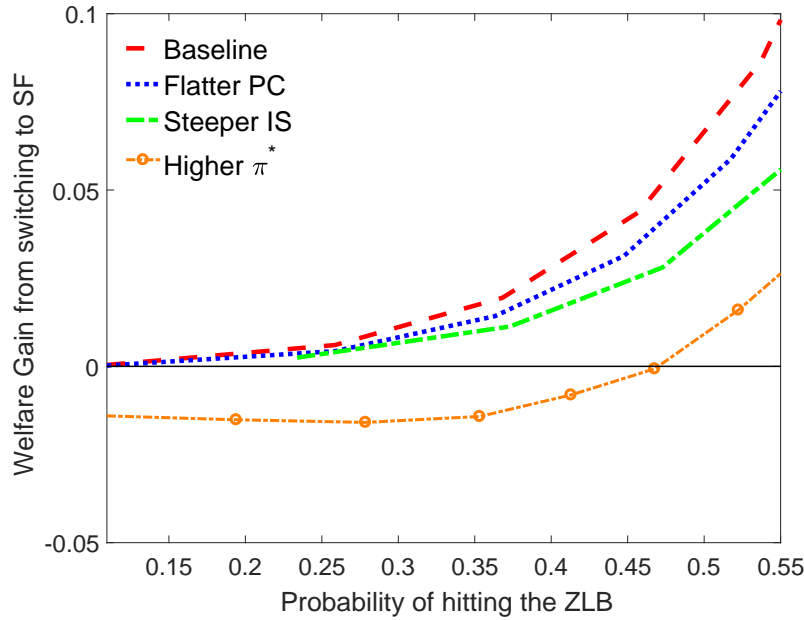
Figure 8: Welfare Gain - Demand shocks

of shocks. On the y-axis it is shown the welfare benefit of moving from a dual mandate to "shortfalls" strategy (i.e. the difference in the loss corresponding to the two strategies).

There are three interesting results. First, moving to a "shortfalls" strategy increases welfare even when the probability of hitting the ZLB is small. This is true for both demand and supply shocks and, in our baseline calibration, moving to a "shortfalls" strategy is welfare enhancing. The gains are obviously bigger the higher the probability of hitting the ZLB. Second, the welfare gains from moving from a "deviations" to a "shortfalls" are still positive but lower when the PC is flatter and when the IS curve is steeper. Third, increasing π^* can make a "shortfalls" strategy detrimental when the probability of hitting the ZLB is low.

These results, to be taken as illustrative, highlight the crucial role of inflation expectations, especially for the effectiveness and welfare consequences of monetary strategies.

4 Conclusion

How does this very simple model help us understand the FOMC's decision to switch from "deviations" to "shortfalls" and what are the costs and benefits?

The model makes clear where the problem lies - the interaction between maximising a symmetric loss function (i.e. treating positive and negative deviations of inflation from

the target on equal footing) under discretion and the increased probability of hitting the ZLB when the natural rate of interest is low. This is particularly a problem for a CB with a dual mandate in the presence of demand shocks. With a symmetric loss function, a dual mandate Central Bank cannot counteract below-target inflation expectations by overheating the labour market. A strict inflation targeter does not face this specific problem but cannot escape from the deflationary sub-optimal equilibrium either. A modestly higher inflation target does not resolve the problem as the promised additional inflation when the ZLB doesn't bind is not sufficient to drag up unconditional inflation expectations by enough to escape the sub-optimal equilibrium and therefore even though realised inflation increases, it is not enough to match the new inflation target. However, notice that under our assumption of a uniform distribution of shocks, a large enough increase in the inflation target will work.

It is arguable whether in practice a dual mandate Central Bank would tighten policy with inflation and inflation expectations below target solely to prevent an unemployment rate that is too low. It is possible this is a pseudo-problem created by the way economists model the loss function rather than a genuine issue. Nevertheless, the shift from referring to "deviations" to focusing on "shortfalls" from maximum employment can be seen as a way of avoiding this possibility. It is also equivalent to a *de facto* increase in the inflation target. As has been widely commented, this change of language introduces a "doveish" bias to monetary policy. This helps counteract the downward bias of the ZLB and with appropriate calibration can help to deliver the inflation target on average. As a result, shifting to a "shortfalls" strategy could be sufficient. However, as noted in the text, the outcomes on unemployment are not actually more doveish under the "shortfalls" strategy. The risk with this strategy is that the easing bias is less and less necessary if the natural rate of interest increases. In the limit, when the ZLB doesn't bind, it becomes the discretionary inflation bias that was highlighted in the earlier literature (Barro and Gordon, 1983).

Flexible average inflation targeting is in some ways an equivalent strategy to "shortfalls" because it raises the *de facto* inflation target when the ZLB doesn't bind. The main difference is that it is time varying and becomes less and less relevant if inflation converges on the target. But when the natural rate of interest is particularly low, the effect of both strategies together is essentially additive. This is an advantage in circumstances in which neither strategy alone would be enough to escape the sub-optimal equilibrium. However, both together could become a victim of their own success, particularly if the natural rate of interest rises again because inflation expectations could become entrenched above the target as a result of the easing bias. It is therefore sensible that the FOMC has

decided to review its strategy every five years.

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Appendix

A Derivations

The economy is described by a standard new-classical Phillips curve

$$u = u_N - a(\pi - \pi^e) - s$$

and an IS curve

$$y - y_N = -b\left(i - \pi^e - r^*\right) - d$$

where u and u_N are respectively unemployment and its natural rate, π is the inflation rate, π^e is the inflation expectation and s is a supply shock. i is the nominal interest rate, r^* is the natural rate of interest and d is a demand shock. Assuming that the Okun's law holds, $y - y_N = -\omega\left(u - u_N\right)$, and for simplicity setting $\omega = 1$, the IS curve becomes

$$u = u_N + b\left(i - \pi^e - r^*\right) - d$$

We now are interested in understanding the implication of an optimal interest rate rule set by a Central Bank (CB), focusing on how its mandate and the source of fluctuation affect the evolution of inflation in the economy. We present four different derivations.

A.1 Dual Mandate

We start by assuming that the CB has a dual mandate in deviations of inflation and unemployment from their target level (it can be shown that this can be derived from a quadratic approximation of the utility of the representative household):

$$Loss = \frac{1}{2}\left(\pi - \pi^*\right)^2 + \frac{\lambda}{2}\left(u - u_N\right)^2$$

Notice that this formulation, by setting $\lambda = 0$ will allow us to consider a perfect inflation targeter CB. We now consider choices of monetary policy that minimize the loss function subject to the sequence of constraints given by the Phillips curve. Indeed, equilibrium paths must also satisfy the IS curve, however, in the present case, the optimal paths of inflation and output can be determined without reference to the IS. We will after make sure that the implied path for nominal interest rate is always non negative.

We can therefore write the Lagrangian

$$\pounds = \frac{1}{2}(\pi - \pi^*)^2 + \frac{\lambda}{2}(u - u_N)^2 + \xi\left[u - u_N + a(\pi - \pi^e)\right]$$

and minimize with respect to π and u :

$$\text{FOC} \quad [\pi] \quad (\pi - \pi^*) + a\xi = 0$$
$$\text{FOC} \quad [u] \quad \lambda(u - u_N) + \xi = 0$$

which combined gives us the path of inflation and output minimizing the cost of inflation and output variability:

$$(u - u_N) = \frac{1}{a\lambda}(\pi - \pi^*)$$

For better understanding of the mechanisms, we first assume that we live in a world with only demand shocks $(s = 0)$ and then move to a world with only supply shocks $(d = 0)$

Demand shocks only

Substitute the optimal path in the Phillips curve

$$\pi - \pi^e = -\frac{1}{a^2\lambda}(\pi - \pi^*)$$

to get the unconstrained inflation path:

$$\pi = \frac{a^2\lambda}{a^2\lambda + 1}\pi^e + \frac{1}{a^2\lambda + 1}\pi^*$$

Now, add $-\pi^*$ from both sides to get

$$\pi - \pi^* = \frac{a^2\lambda}{a^2\lambda + 1}(\pi^e - \pi^*)$$

which, substituted into the optimal path gives us

$$u - u_N = \frac{1}{a\lambda}\left[\frac{a^2\lambda}{a^2\lambda + 1}(\pi^e - \pi^*)\right] = \frac{a}{a^2\lambda + 1}(\pi^e - \pi^*)$$

Now we need to check if the desired path of inflation and output gap is consistent with an implied path of the nominal interest rate which is always non-negative. Therefore, we

take the IS curve and substitute the optimal path of unemployment to derived the implied nominal interest rate

$$i = \frac{1}{b}(u - u_N) + \pi^e + r^* + \frac{1}{b}d = \frac{a}{b(a^2\lambda + 1)}(\pi^e - \pi^*) + \pi^e + r^* + \frac{1}{b}d =$$

$$= \frac{a + b(a^2\lambda + 1)}{b(a^2\lambda + 1)}\pi^e - \frac{a}{b(a^2\lambda + 1)}\pi^* + r^* + \frac{1}{b}d$$

Now, given that nominal interest rates are constrained at zero, the policy rule becomes

$$i = \max\left(0, \frac{a + b(a^2\lambda + 1)}{b(a^2\lambda + 1)}\pi^e - \frac{a}{b(a^2\lambda + 1)}\pi^* + r^* + \frac{1}{b}d\right)$$

which will imply that the monetary policy might be hitting the zero lower bound (ZLB) depending on the demand shock. This gives us a threshold demand shock $\bar{d}$ for which the ZLB would be binding:

$$\bar{d} = \frac{a}{(a^2\lambda + 1)}\pi^* - \frac{[a + b(a^2\lambda + 1)]}{(a^2\lambda + 1)}\pi^e - br^* < d$$

Therefore,

$$\begin{cases} i = \frac{a+b(a^2\lambda+1)}{b(a^2\lambda+1)}\pi^e - \frac{a}{b(a^2\lambda+1)}\pi^* + r^* + \frac{1}{b}d \quad \text{and} \quad \pi = \frac{a^2\lambda}{a^2\lambda+1}\pi^e + \frac{1}{a^2\lambda+1}\pi^* & \text{if } d > \bar{d} \\ i = 0 & \text{if } d < \bar{d} \end{cases}$$

It is left to determine what would be the inflation path when the nominal interest rate is constrained by the ZLB. That is easily determined by combining the Phillips and the IS curve, obtaining:

$$0 = \frac{1}{b}[-a(\pi - \pi^e)] + \pi^e + r^* + \frac{1}{b}d = -\frac{a}{b}\pi + \frac{a + b}{b}\pi^e + r^* + \frac{1}{b}d =$$

$$= -a\pi + (a + b)\pi^e + br^* + d$$

therefore

$$\pi_{ZLB} = \frac{(a + b)}{a}\pi^e + \frac{b}{a}r^* + \frac{1}{a}d$$

Assume that the demand shock d is uniformly distributed between $[d_L, d_H]$. We can

compute expected inflation:

$$E(\pi) = \pi^e = \int_{d_L}^{\bar{d}} \pi_{ZLB} p(d) d(d) + \int_{\bar{d}}^{d_H} \pi_{NZLB} p(d) d(d)$$

$$= \int_{d_L}^{\bar{d}} \left(\frac{(a+b)}{a} \pi^e + \frac{b}{a} r^* + \frac{1}{a} d \right) p(d) d(d) + \int_{\bar{d}}^{d_H} \left(\frac{a^2\lambda}{a^2\lambda+1} \pi^e + \frac{1}{a^2\lambda+1} \pi^* \right) p(d) d(d)$$

$$= \left(\frac{(a+b)}{a} \pi^e + \frac{b}{a} r^* \right) \frac{1}{d_H - d_L} \int_{d_L}^{\bar{d}} d(d) + \frac{1}{a} \frac{1}{d_H - d_L} \int_{d_L}^{\bar{d}} d\, d(d) + \left(\frac{a^2\lambda}{a^2\lambda+1} \pi^e + \frac{1}{a^2\lambda+1} \pi^* \right) \frac{1}{d_H - d_L} \int_{\bar{d}}^{d_H} d(d)$$

$$(d_H - d_L) \pi^e = \left(\frac{(a+b)}{a} \pi^e + \frac{b}{a} r^* \right) \left(\bar{d} - d_L \right) + \frac{1}{a} \left[d^2 \right]_{d_L}^{\bar{d}} + \left(\frac{a^2\lambda}{a^2\lambda+1} \pi^e + \frac{1}{a^2\lambda+1} \pi^* \right) \left(d_H - \bar{d} \right)$$

$$(d_H - d_L) \pi^e = \frac{(a+b)}{a} \bar{d} \pi^e + \frac{b}{a} r^* \bar{d} - \frac{(a+b) d_L}{a} \pi^e + \frac{1}{2a} \bar{d}^2 + \frac{a^2\lambda d_H}{a^2\lambda+1} \pi^e - \frac{a^2\lambda}{a^2\lambda+1} \pi^e \bar{d} - \frac{\pi^*}{a^2\lambda+1} \bar{d} + X$$

where X groups and renames the constant values

$$X = -\frac{br^* d_L}{a} - \frac{1}{2a} d_L^2 + \frac{\pi^*}{a^2\lambda+1} d_H$$

We can therefore rewrite it as

$$(d_H - d_L) \pi^e = \left[\frac{(a+b)}{a} - \frac{a^2\lambda}{a^2\lambda+1} \right] \bar{d} \pi^e + \left[\frac{b}{a} r^* - \frac{\pi^*}{a^2\lambda+1} \right] \bar{d} - \left[\frac{(a+b) d_L}{a} - \frac{a^2\lambda d_H}{a^2\lambda+1} \right] \pi^e + \frac{1}{a} \bar{d}^2 + X$$

and then simplify it to

$$(d_H - d_L) \pi^e = \left[\frac{a^3\lambda - a^3\lambda + a + b\left[a^2\lambda+1\right]}{a\left[a^2\lambda+1\right]} \right] \bar{d} \pi^e + \left[\frac{b\left(a^2\lambda+1\right) r^* - a\pi^*}{a\left[a^2\lambda+1\right]} \right] \bar{d} - \left[\frac{\left[a^3\lambda + a + b\left(a^2\lambda+1\right)\right] d_L - a^3\lambda d_H}{a\left[a^2\lambda+1\right]} \right] \pi^e$$

$$0 = \left[\frac{a + b\left[a^2\lambda+1\right]}{a\left[a^2\lambda+1\right]} \right] \bar{d} \pi^e + \left[\frac{b\left(a^2\lambda+1\right) r^* - a\pi^*}{a\left[a^2\lambda+1\right]} \right] \bar{d} +$$

$$- \left[\frac{\left[a^3\lambda + a + b\left(a^2\lambda+1\right)\right] d_L - a^3\lambda d_H + a^3\lambda d_H + a d_H - a^3\lambda d_L - a d_L}{a\left[a^2\lambda+1\right]} \right] \pi^e + \frac{1}{a} \bar{d}^2 + X$$

$$0 = [a + b\left(a^2\lambda+1\right)] \bar{d} \pi^e + \left[b\left(a^2\lambda+1\right) r^* - a\pi^* \right] \bar{d} - \left[b\left(a^2\lambda+1\right) d_L + a d_H \right] \pi^e + \frac{1}{2} \left[a^2\lambda+1 \right] \bar{d}^2 + a\left(a^2\lambda+1\right) X$$

From the threshold definition we can get an expression of inflation expectations as a function of the threshold productivity:

$$\pi^e = \frac{a}{[a + b\left(a^2\lambda+1\right)]} \pi^* - \frac{\left(a^2\lambda+1\right)}{[a + b\left(a^2\lambda+1\right)]} \bar{d} - \frac{b\left(a^2\lambda+1\right)}{[a + b\left(a^2\lambda+1\right)]} r^*$$

Substituting this, we get a quadratic expression which simplifies nicely:

$$0 = -[a + b(a^2\lambda + 1)]\bar{d}\frac{(a^2\lambda + 1)}{[a + b(a^2\lambda + 1)]}\bar{d} + [a + b(a^2\lambda + 1)]\left[\frac{a\pi^* - b(a^2\lambda + 1)r^*}{[a + b(a^2\lambda + 1)]}\right]\bar{d} +$$

$$+ [b(a^2\lambda + 1)r^* - a\pi^*]\bar{d} + \frac{1}{2}[a^2\lambda + 1]\bar{d}^2 + a(a^2\lambda + 1)X +$$

$$+ [b(a^2\lambda + 1)d_L + ad_H]\frac{(a^2\lambda + 1)}{[a + b(a^2\lambda + 1)]}\bar{d} - [b(a^2\lambda + 1)d_L + ad_H]\left[\frac{a}{[a + b(a^2\lambda + 1)]}\pi^* - \frac{b(a^2\lambda + 1)}{[a + b(a^2\lambda + 1)]}r^*\right]$$

$$0 = -(a^2\lambda + 1)\bar{d}^2 + \frac{1}{2}[a^2\lambda + 1]\bar{d}^2 + (a^2\lambda + 1)\left[\frac{b(a^2\lambda + 1)d_L + ad_H}{[a + b(a^2\lambda + 1)]}\right]\bar{d} + Y + a(a^2\lambda + 1)X$$

where

$$Y = -\left[b(a^2\lambda + 1)d_L + ad_H\right]\left[\frac{a}{[a + b(a^2\lambda + 1)]}\pi^* - \frac{b(a^2\lambda + 1)}{[a + b(a^2\lambda + 1)]}r^*\right]$$

Our final quadratic expression, in presence of only demand shocks is:

$$0 = -\frac{1}{2}\bar{d}^2 + \frac{b(a^2\lambda + 1)d_L + ad_H}{[a + b(a^2\lambda + 1)]}\bar{d} + \frac{1}{(a^2\lambda + 1)}Y + aX$$

To find a solution, we need to identify

$A = -\frac{1}{2}$

$B = \frac{b(a^2\lambda + 1)d_L + ad_H}{[a + b(a^2\lambda + 1)]}$

$C = \frac{1}{(a^2\lambda + 1)}Y + aX$

and notice that $\bar{d} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$ has only one stable root.

Supply shocks only

We then proceed to check what happens when $(s \neq 0)$ and $(d = 0)$. Up to the optimal path, the derivations are the same but now, in determining the path of inflation, supply shocks will appear into the equation:

Substitute the optimal path in the Phillips curve to get

$$\pi - \pi^e = -\frac{1}{a^2\lambda}(\pi - \pi^*) - \frac{1}{a}s$$

which gives

$$\pi = \frac{a^2\lambda}{a^2\lambda + 1}\pi^e + \frac{1}{a^2\lambda + 1}\pi^* - \frac{a\lambda}{a^2\lambda + 1}s$$

Now add and subtract π^* to get

$$\pi - \pi^* = \pi^e - \frac{1}{a^2\lambda}(\pi - \pi^*) - \pi^* - \frac{1}{a}s$$

$$\pi - \pi^* = \frac{a^2\lambda}{a^2\lambda + 1}\left(\pi^e - \pi^* - \frac{1}{a}s\right)$$

which allows us to derive the path of the unemployment gap using the optimal path derived from minimizing the square deviations.

$$u - u_N = \frac{1}{a\lambda}\left[\frac{a^2\lambda}{a^2\lambda + 1}\left(\pi^e - \pi^* - \frac{1}{a}s\right)\right] = \frac{a}{a^2\lambda + 1}\left(\pi^e - \pi^* - \frac{1}{a}s\right)$$

As before, now we need to make sure for what values of the supply shock the nominal interest rate is unconstrained:

$$i = \frac{a}{b(a^2\lambda + 1)}\left(\pi^e - \pi^* - \frac{1}{a}s\right) + \pi^e + r^* = \frac{a + b(a^2\lambda + 1)}{b(a^2\lambda + 1)}\pi^e - \frac{a}{b(a^2\lambda + 1)}\pi^* - \frac{1}{b(a^2\lambda + 1)}s + r^*$$

Given the presence of the ZLB then, we will have a interest rate rule of the following form:

$$i = \max\left(0, \frac{a + b(a^2\lambda + 1)}{b(a^2\lambda + 1)}\pi^e - \frac{a}{b(a^2\lambda + 1)}\pi^* - \frac{1}{b(a^2\lambda + 1)}s + r^*\right)$$

Which will give us a threshold on the realization of the supply shock that will make the ZLB binding

$$\frac{a + b(a^2\lambda + 1)}{b(a^2\lambda + 1)}\pi^e - \frac{a}{b(a^2\lambda + 1)}\pi^* + r^* > \frac{1}{b(a^2\lambda + 1)}s$$

$$\bar{s} = \left[a + b(a^2\lambda + 1)\right]\pi^e - a\pi^* + b(a^2\lambda + 1)r^* > s$$

$$\begin{cases} i = \dfrac{a+b\left(a^2\lambda+1\right)}{b\left(a^2\lambda+1\right)}\pi^e - \dfrac{a}{b\left(a^2\lambda+1\right)}\pi^* - \dfrac{1}{b\left(a^2\lambda+1\right)}s + r^* & if\,s < \bar{s} \\ \text{and} \quad \pi = \dfrac{a^2\lambda}{a^2\lambda+1}\pi^e + \dfrac{1}{a^2\lambda+1}\pi^* - \dfrac{a\lambda}{a^2\lambda+1}s & \\ i = 0 & if\,s > \bar{s} \end{cases}$$

Now it is left to determine the inflation rate when the ZLB would be binding. Combining the Phillips curve with the constrained IS curve

$$0 = \frac{1}{b}\left[-a\left(\pi - \pi^e\right) - s\right] + \pi^e + r^* = -\frac{a}{b}\pi + \frac{a+b}{b}\pi^e - \frac{1}{b}s + r^* =$$
$$= -a\pi + (a+b)\,\pi^e - s + br^*$$

which gives

$$\pi_{ZLB} = \frac{(a+b)}{a}\pi^e - \frac{1}{a}s + \frac{b}{a}r^*$$

Now, as before, let's assume that s is uniformly distributed between $[s_L, s_H]$. We can compute expected inflation:

$$E(\pi) = \pi^e = \int_{s_L}^{\bar{s}} \pi_{NZLB}\,p(s)ds + \int_{\bar{s}}^{s_H} \pi_{ZLB}\,p(s)ds$$

$$\pi^e = \int_{s_L}^{\bar{s}} \left(\frac{a^2\lambda}{a^2\lambda+1}\pi^e + \frac{1}{a^2\lambda+1}\pi^* - \frac{a\lambda}{a^2\lambda+1}s\right)p(s)ds + \int_{\bar{s}}^{s_H}\left(\frac{(a+b)}{a}\pi^e - \frac{1}{a}s + \frac{b}{a}r^*\right)p(s)ds$$

$$\pi^e = \left(\frac{a^2\lambda}{a^2\lambda+1}\pi^e + \frac{1}{a^2\lambda+1}\pi^*\right)\frac{1}{s_H - s_L}\int_{s_L}^{\bar{s}}ds - \frac{a\lambda}{a^2\lambda+1}\frac{1}{s_H - s_L}\int_{s_L}^{\bar{s}}s\,ds +$$
$$+ \left(\frac{(a+b)}{a}\pi^e + \frac{b}{a}r^*\right)\frac{1}{s_H - s_L}\int_{\bar{s}}^{s_H}ds - \frac{1}{a}\frac{1}{s_H - s_L}\int_{\bar{s}}^{s_H}s\,ds$$

$$(s_H - s_L)\,\pi^e = \left(\frac{a^2\lambda}{a^2\lambda+1}\pi^e + \frac{1}{a^2\lambda+1}\pi^*\right)(\bar{s} - s_L) - \frac{a\lambda}{2\left[a^2\lambda+1\right]}\left[s^2\right]_{s_L}^{\bar{s}} +$$
$$+ \left(\frac{(a+b)}{a}\pi^e + \frac{b}{a}r^*\right)(s_H - \bar{s}) - \frac{1}{2a}\left[s^2\right]_{\bar{s}}^{s_H}$$

$$(s_H - s_L)\,\pi^e = \frac{a^2\lambda}{a^2\lambda+1}\bar{s}\pi^e - \frac{\left(a^2\lambda\right)s_L}{a^2\lambda+1}\pi^e + \frac{\pi^*}{a^2\lambda+1}\bar{s} - \frac{a\lambda}{2\left[a^2\lambda+1\right]}\bar{s}^2 +$$
$$+ \frac{(a+b)s_H}{a}\pi^e - \frac{(a+b)}{a}\bar{s}\pi^e - \frac{br^*}{a}\bar{s} + \frac{1}{2a}\bar{s}^2 + X$$

where X groups and renames the constant values

$$X = -\frac{s_L}{a^2\lambda + 1}\pi^* + \frac{a\lambda}{2\left[a^2\lambda + 1\right]}s_L^2 + \frac{bs_H}{a}r^* - \frac{1}{2a}s_H^2$$

which we can rewrite as

$$(s_H - s_L)\,\pi^e = \left[\frac{a^2\lambda}{a^2\lambda + 1} - \frac{(a+b)}{a}\right]\bar{s}\pi^e + \left[\frac{(a+b)\,s_H}{a} - \frac{\left(a^2\lambda\right)s_L}{a^2\lambda + 1}\right]\pi^e +$$

$$+ \left[\frac{\pi^*}{a^2\lambda + 1} - \frac{br^*}{a}\right]\bar{s} + \left[\frac{1}{2a} - \frac{a\lambda}{2\left[a^2\lambda + 1\right]}\right]\bar{s}^2 + X$$

and then simplify as

$$(s_H - s_L)\,\pi^e = \left[\frac{a^3\lambda - a^3\lambda - a - b\left[a^2\lambda + 1\right]}{a\left[a^2\lambda + 1\right]}\right]\bar{s}\pi^e + \left[\frac{\left[a^3\lambda + a + b\left(a^2\lambda + 1\right)\right]s_H - a^3\lambda s_L}{a\left[a^2\lambda + 1\right]}\right]\pi^e +$$

$$+ \left[\frac{a\pi^* - b\left(a^2\lambda + 1\right)r^*}{a\left[a^2\lambda + 1\right]}\right]\bar{s} + \left[\frac{a^2\lambda + 1 - a^2\lambda}{2a\left[a^2\lambda + 1\right]}\right]\bar{s}^2 + X$$

$$0 = \left[\frac{-a - b\left[a^2\lambda + 1\right]}{a\left[a^2\lambda + 1\right]}\right]\bar{s}\pi^e + \left[\frac{a^3\lambda s_H + a s_H + b\left(a^2\lambda + 1\right)s_H - a^3\lambda s_L - a^3\lambda s_H - a s_H + a^3\lambda s_L + a s_L}{a\left[a^2\lambda + 1\right]}\right]\pi^e +$$

$$+ \left[\frac{a\pi^* - b\left(a^2\lambda + 1\right)r^*}{a\left[a^2\lambda + 1\right]}\right]\bar{s} + \left[\frac{1}{2a\left[a^2\lambda + 1\right]}\right]\bar{s}^2 + X$$

$$0 = -[a + b\left(a^2\lambda + 1\right)]\bar{s}\pi^e + \left[b\left(a^2\lambda + 1\right)s_H + a s_L\right]\pi^e + \left[a\pi^* - b\left(a^2\lambda + 1\right)r^*\right]\bar{s} + \frac{1}{2}\bar{s}^2 + a\left(a^2\lambda + 1\right)X$$

Using the threshold definition, we can write inflation expectations as a function of the threshold supply shock:

$$\pi^e = \frac{1}{\left[a + b\left(a^2\lambda + 1\right)\right]}\bar{s} + \frac{a}{\left[a + b\left(a^2\lambda + 1\right)\right]}\pi^* - \frac{b\left(a^2\lambda + 1\right)}{\left[a + b\left(a^2\lambda + 1\right)\right]}r^*$$

then, trasforming the previous expression in

$$0 = -[a + b\left(a^2\lambda + 1\right)]\frac{1}{\left[a + b\left(a^2\lambda + 1\right)\right]}\bar{s}^2 - [a + b\left(a^2\lambda + 1\right)]\left[\frac{a}{\left[a + b\left(a^2\lambda + 1\right)\right]}\pi^* - \frac{b\left(a^2\lambda + 1\right)}{\left[a + b\left(a^2\lambda + 1\right)\right]}r^*\right]\bar{s} +$$

$$+ \left[b\left(a^2\lambda + 1\right)s_H + a s_L\right]\frac{1}{\left[a + b\left(a^2\lambda + 1\right)\right]}\bar{s} + \left[b\left(a^2\lambda + 1\right)s_H + a s_L\right]\left[\frac{a}{\left[a + b\left(a^2\lambda + 1\right)\right]}\pi^* - \frac{b\left(a^2\lambda + 1\right)}{\left[a + b\left(a^2\lambda + 1\right)\right]}r^*\right] +$$

$$+ \left[a\pi^* - b\left(a^2\lambda + 1\right)r^*\right]\bar{s} + \frac{1}{2}\bar{s}^2 + a\left(a^2\lambda + 1\right)X$$

which simplifies to

$$0 = -\bar{s}^2 + \frac{1}{2}\bar{s}^2 + \left[b\left(a^2\lambda + 1\right) r^* - a\pi^* + \frac{b\left(a^2\lambda + 1\right) s_H + as_L}{[a + b\left(a^2\lambda + 1\right)]} + a\pi^* - b\left(a^2\lambda + 1\right) r^* \right] \bar{s} + Y + a\left(a^2\lambda + 1\right) X$$

where

$$Y = \left[b\left(a^2\lambda + 1\right) s_H + as_L \right] \left[\frac{a}{[a + b\left(a^2\lambda + 1\right)]}\pi^* - \frac{b\left(a^2\lambda + 1\right)}{[a + b\left(a^2\lambda + 1\right)]}r^* \right]$$

The final expression for the quadratic equation will be

$$0 = -\frac{1}{2}\bar{s}^2 + \frac{b\left(a^2\lambda + 1\right) s_H + as_L}{[a + b\left(a^2\lambda + 1\right)]}\bar{s} + Y + a\left(a^2\lambda + 1\right) X$$

where, to find a solution of the quadratic equation we will have

$A = -\frac{1}{2}$

$B = \frac{b\left(a^2\lambda+1\right)s_H + as_L}{[a+b\left(a^2\lambda+1\right)]}$

$C = Y + a\left(a^2\lambda + 1\right) X$

As before, for the supply shock, the solution $\bar{s} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$ has only one stable root.

A.2   Shortfall Mandate

We now move to assume that the CB has a different loss function and cares only about the level of unemployment and not its deviation from the natural level, implying a loss function of the following form:

$$Loss = \frac{1}{2}\left(\pi - \pi^*\right)^2 + \phi\left(u - u_N\right)$$

Writing the Lagrangian

$$\pounds = \frac{1}{2}\left(\pi - \pi^*\right)^2 + \phi\left(u - u_N\right) + \xi\left[u - u_N + a\left(\pi - \pi^e\right)\right]$$

we get

$$\text{FOC}\quad [\pi]\qquad \left(\pi - \pi^*\right) + a\xi = 0$$
$$\text{FOC}\quad [u]\qquad \phi + \xi = 0$$

that combined gives us $a\phi = (\pi - \pi^*)$, which is the optimal rule that can be rewritten as:

$$\pi = a\phi + \pi^*$$

Now, as before, this gives us the path of inflation minimizing the loss function and it will not depend on the output gap. We investigate two separate cases. One in which there are only demand shock $(s = 0)$ and one in which there are only supply shocks $(d = 0)$.

Demand shock

When unconstrained, $\pi = a\phi + \pi^*$, which, substituted in the combination between the IS and Phillips curve, will give us the path of the nominal interest rate necessary to satisfy this rule.

$$-a(\pi - \pi^e) = b(i - \pi^e - r^*) - d$$

$$i = \frac{a+b}{b}\pi^e + r^* + \frac{1}{b}d - \frac{a}{b}\pi = \frac{a+b}{b}\pi^e + r^* + \frac{1}{b}d - \frac{a^2\phi}{b} - \frac{a}{b}\pi^*$$

which gives us a nominal interest rate path

$$i = \max\left(0, \frac{a+b}{b}\pi^e + r^* + \frac{1}{b}d - \frac{a^2\phi}{b} - \frac{a}{b}\pi^*\right)$$

This will imply that the monetary policy might be hitting the zero lower bound (ZLB) depending on the demand shock. This gives us a threshold demand shock $\bar{d}$ for which the ZLB would be binding:

$$-\frac{a+b}{b}\pi^e - r^* + \frac{a^2\phi}{b} + \frac{a}{b}\pi^* < \frac{1}{b}d$$

$$\bar{d} = -(a+b)\pi^e - br^* + a^2\phi + a\pi^* < d$$

Therefore,

$$\begin{cases} i = \frac{a+b}{b}\pi^e + r^* + \frac{1}{b}d - \frac{a^2\phi}{b} - \frac{a}{b}\pi^* \quad \text{and} \quad \pi = +\pi^* \quad if\, d > \bar{d} \\ i = 0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad if\, d < \bar{d} \end{cases} \quad (12)$$

It is left to determine what would be the inflation path when the nominal interest rate is constrained by the ZLB. That is the same we shown before determined by combining

the Phillips and the IS curve, obtaining

$$\pi_{ZLB} = \frac{(a+b)}{a}\pi^e + \frac{b}{a}r^* + \frac{1}{a}d$$

Assume that the demand shock d is uniformly distributed between $[d_L, d_H]$. We can compute expected inflation:

$$E(\pi) = \pi^e = \int_{d_L}^{\bar{d}} \pi_{ZLB} p(d)d(d) + \int_{\bar{d}}^{d_H} \pi_{NZLB} p(d)d(d)$$

$$\pi^e = \int_{d_L}^{\bar{d}} \left(\frac{(a+b)}{a}\pi^e + \frac{b}{a}r^* + \frac{1}{a}d\right) p(d)d(d) + \int_{\bar{d}}^{d_H} (a\phi + \pi^*) p(d)d(d)$$

$$\pi^e = \left(\frac{(a+b)}{a}\pi^e + \frac{b}{a}r^*\right) \frac{1}{d_H - d_L} \int_{d_L}^{\bar{d}} d(d) + \frac{1}{a}\frac{1}{d_H - d_L} \int_{d_L}^{\bar{d}} dd(d) + (a\phi + \pi^*) \frac{1}{d_H - d_L} \int_{\bar{d}}^{d_H} d(d)$$

$$(d_H - d_L)\pi^e = \left(\frac{(a+b)}{a}\pi^e + \frac{b}{a}r^*\right)\left(\bar{d} - d_L\right) + \frac{1}{a}\left[d^2\right]_{d_L}^{\bar{d}} + (a\phi + \pi^*)\left(d_H - \bar{d}\right)$$

$$(d_H - d_L)\pi^e = \frac{(a+b)}{a}\bar{d}\pi^e + \frac{b}{a}r^*\bar{d} - \frac{(a+b)d_L}{a}\pi^e + \frac{1}{2a}\bar{d}^2 - (a\phi + \pi^*)\bar{d} + X$$

where X groups and renames the constant values

$$X = -\frac{br^*d_L}{a} - \frac{1}{2a}d_L^2 + a\phi d_H + \pi^* d_H$$

We can simplify it as

$$0 = \frac{(a+b)}{a}\bar{d}\pi^e + \left[\frac{br^* - a^2\phi - a\pi^*}{a}\right]\bar{d} - \left[\frac{ad_L + bd_L + ad_H - ad_L}{a}\right]\pi^e + \frac{1}{2a}\bar{d}^2 + X$$

$$0 = (a+b)\bar{d}\pi^e + \left[br^* - a^2\phi - a\pi^*\right]\bar{d} - [bd_L + ad_H]\pi^e + \frac{1}{2}\bar{d}^2 + aX$$

From the threshold definition we can get an expression of inflation expectations as a function of the threshold shock:

$$\pi^e = \frac{a}{[a+b]}\pi^* - \frac{1}{[a+b]}\bar{d} - \frac{b}{[a+b]}r^* + \frac{a^2\phi}{[a+b]}$$

Substituting this, we get a quadratic expression which simplifies nicely:

$$0 = -(a+b)\frac{1}{[a+b]}\bar{d}^2 + (a+b)\left[\frac{a\pi^* - br^* + a^2\phi}{[a+b]}\right]\bar{d} + \left[br^* - a^2\phi - a\pi^*\right]\bar{d} - \frac{[bd_L + ad_H]}{[a+b]}\bar{d} + \frac{1}{2}\bar{d}^2 + Y + aX$$

where

$$Y = -[bd_L + ad_H] \left[\frac{a\pi^* - br^* + a^2\phi}{[a+b]} \right]$$

gives us our final quadratic expression:

$$0 = -\frac{1}{2}\bar{d}^2 + -\frac{[bd_L + ad_H]}{[a+b]}\bar{d} + Y + aX$$

To find a solution, we need to identify

$A = -\dfrac{1}{2}$

$B = \dfrac{[bd_L + ad_H]}{[a+b]}$

$C = Y + aX$

Notice that $\bar{d} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$ has only one stable root.

Shortfall Supply

When unconstrained, $\pi = a\lambda + \pi^*$, which, substituted in the combination between the IS and Phillips curve, will give us the path of the nominal interest rate necessary to satisfy this rule:

$$-a(\pi - \pi^e) - s = b(i - \pi^e - r^*)$$
$$i = r^* + \frac{1}{b}[(a+b)\pi^e - a\pi] - \frac{1}{b}s$$
$$i = r^* + \frac{1}{b}\left[(a+b)\pi^e - a^2\phi - a\pi^*\right] - \frac{1}{b}s$$
$$i = \max\left(0, r^* + \frac{1}{b}\left[(a+b)\pi^e - a^2\phi - a\pi^*\right] - \frac{1}{b}s\right)$$

Which will give us a threshold on the realization of the supply shock that will make the ZLB binding

$$r^* + \frac{1}{b}\left[(a+b)\pi^e - a^2\phi - a\pi^*\right] > \frac{1}{b}s$$

$$overlines = br^* + (a+b)\pi^e - a^2\phi - a\pi^* > s$$

$$\begin{cases} i = r^* + \frac{1}{b}\left[(a+b)\pi^e - a^2\phi - a\pi^*\right] - \frac{1}{b}s \text{ and } & if\, s < \bar{s} \\ i = 0 & if\, s > \bar{s} \end{cases} \tag{13}$$

and in the second expression we are back to the previously seen case of

$$\pi_{ZLB} = \frac{(a+b)}{a}\pi^e - \frac{1}{a}s + \frac{b}{a}r^*$$

Now, as before, let's assume that s is uniformly distributed between $[s_L, s_H]$. We can compute expected inflation:

$$E(\pi) = \pi^e = \int_{s_L}^{\bar{s}} \pi_{NZLB}p(s)ds + \int_{\bar{s}}^{s_H} \pi_{ZLB}p(s)ds$$

$$\pi^e = \int_{s_L}^{\bar{s}} (a\phi + \pi^*)\, p(s)ds + \int_{\bar{s}}^{s_H} \left(\frac{(a+b)}{a}\pi^e - \frac{1}{a}s + \frac{b}{a}r^*\right)p(s)ds$$

$$\pi^e = (a\phi + \pi^*)\frac{1}{s_H - s_L}\int_{s_L}^{\bar{s}} ds + \left(\frac{(a+b)}{a}\pi^e + \frac{b}{a}r^*\right)\frac{1}{s_H - s_L}\int_{\bar{s}}^{s_H} ds - \frac{1}{a}\frac{1}{s_H - s_L}\int_{\bar{s}}^{s_H} sds$$

$$(s_H - s_L)\pi^e = (a\phi + \pi^*)(\bar{s} - s_L) + \left(\frac{(a+b)}{a}\pi^e + \frac{b}{a}r^*\right)(s_H - \bar{s}) - \frac{1}{2a}\left[s^2\right]_{\bar{s}}^{s_H}$$

$$(s_H - s_L)\pi^e = \left(\frac{a^2\phi + a\pi^* - br^*}{a}\right)\bar{s} + \frac{(a+b)s_H}{a}\pi^e - \frac{(a+b)}{a}\bar{s}\pi^e + \frac{1}{2a}\bar{s}^2 + X$$

where X groups and renames the constant values

$$X = -(a\phi + \pi^*)s_L + \frac{br^*}{a}s_H - \frac{1}{2a}s_H^2$$

which we he rewritten as

$$0 = \left(\frac{a^2\phi + a\pi^* - br^*}{a}\right)\bar{s} + \left[\frac{(a+b)s_H - as_H + as_L}{a}\right]\pi^e - \frac{(a+b)}{a}\bar{s}\pi^e + \frac{1}{2a}\bar{s}^2 + X$$

$$0 = \left(a^2\phi + a\pi^* - br^*\right)\bar{s} + [bs_H + as_L]\pi^e - (a+b)\bar{s}\pi^e + \frac{1}{2}\bar{s}^2 + aX$$

Using the threshold definition, we can write inflation expectations as a function of the threshold supply shock:

$$\pi^e = \frac{a^2\phi - br^* + a\pi^*}{(a+b)} + \frac{1}{(a+b)}\bar{s}$$

then, transforming the previous expression in

$$0 = \left(a^2\phi + a\pi^* - br^*\right)\bar{s} + [bs_H + as_L]\frac{a^2\phi - br^* + a\pi^*}{(a+b)} + \frac{[bs_H + as_L]}{(a+b)}\bar{s} - (a+b)\bar{s}\left[\frac{a^2\phi - br^* + a\pi^*}{(a+b)}\right] - \bar{s}^2 + \frac{1}{2}\bar{s}^2 + aX$$

which simplifies to

$$0 = -\frac{1}{2}\bar{s}^2 + \frac{[bs_H + as_L]}{(a+b)}\bar{s} + Y + aX$$

where

$$Y = [bs_H + as_L]\frac{a^2\phi - br^* + a\pi^*}{(a+b)}$$

The final expression for the quadratic equation will be

$$0 = -\frac{1}{2}\bar{s}^2 + \frac{[bs_H + as_L]}{(a+b)}\bar{s} + Y + aX$$

where, to find a solution to the quadratic equation we will have

$$A = -\frac{1}{2}$$
$$B = \frac{[bs_H + as_L]}{(a+b)}$$
$$C = Y + aX$$

As before, for the supply shock, the solution $\bar{s} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$ has only one stable root.